# Heavy sterile neutrinos: Bounds from big-bang nucleosynthesis and SN 1987A

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## Abstract

Cosmological and astrophysical effects of heavy (10–200 MeV) sterile Dirac neutrinos, mixed with the active ones, are considered. The bounds on mass and mixing angle from both supernovae and big-bang nucleosynthesis are presented.

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# 1  Introduction

There are strong and well known indications that the three ordinary (active) neutrinos $\nu_a$ ($a = e,\,\mu,\,\tau$) are mixtures of different mass eigenstates $\nu_j$, with $j = 1, 2, 3, \ldots$ (for a review see e.g. [1]). Active neutrinos can mix among themselves, and possibly *only* among themselves, in which case $j = 1, 2, 3$. However, it is also possible that there are mixings of the active neutrinos with new sterile ones, $\nu_s$, that do not have any direct coupling to $W$ and $Z$ bosons mediating weak interactions. Sterile neutrinos have repeatedly been suggested as solutions of various anomalies in neutrino experiments. In particular, if all the present day data are correct, then at least one sterile neutrino must participate. For an explanation of the known neutrino anomalies, the mass differences should be very small, it should lie in the eV range or even in the sub-eV range. However, if we admit that there may be new neutrino states mixed to the usual ones, we should keep an open mind to possible values of their masses. It is an interesting question which values of masses and mixing angles can be excluded by direct experiments as well as by cosmology and astrophysics. The last two topics are addressed in the present paper which was partly stimulated by the recent limits on mixing of $\nu_\tau$ with a heavy sterile neutrino found by the NOMAD collaboration [2], and partly by our paper [3] where we have found that cosmology and astrophysics practically exclude the interpretation of the KARMEN anomaly [4] by a 33.9 MeV neutrino mixed with $\nu_\tau$ (astrophysical and cosmological limits on 33.9 MeV sterile neutrino were also considered in ref. [5]). According to a statement of the KARMEN collaboration made at Neutrino 2000 [6] the anomaly was not observed in upgraded detector KARMEN 2, but the question still remains which area in the mass-mixing-plane can be excluded.

In what follows we will derive the bounds on masses and mixings that follow from big bang nucleosynthesis (BBN) and from the duration of the supernova (SN) 1987A neutrino burst. We essentially repeat our previous work [3], but now lifting the restriction on a specific value of mass and lifetime of the heavy neutrinos and considering mixings with all active states $\nu_e$, $\nu_\mu$ and $\nu_\tau$. We assume that the heavy neutrinos

have Dirac mass. In the case of Majorana mass the limits would be slightly weaker.

## 2  Preliminaries

We assume that the sterile neutrino mixes predominantly with only one active flavour $\nu_a = \nu_e$, $\nu_\mu$ or $\nu_\tau$. The mixed flavour states are expressed in terms of the mass eigenstates and the mixing angle $\theta$ as

$$
\begin{aligned}
\nu_a &= \cos\theta\ \nu_1 + \sin\theta\ \nu_2\,,\\
\nu_s &= -\sin\theta\ \nu_1 + \cos\theta\ \nu_2\,,
\end{aligned}
\tag{1}
$$

where $\nu_1$ and $\nu_2$ are assumed to be the light and heavy mass eigenstates, respectively.

This mixing couples the heavier neutrino to the $Z^0$, allowing for the decay channel

$$
\nu_2 \to \nu_1 + \ell + \bar{\ell}\,,
\tag{2}
$$

where $\nu_1$ is mostly an active flavour and $\ell$ is any lepton with a mass smaller than the mass $m_2$ of the heavy neutrino. We assume that $m_2 < 2m_\mu$ so that the decay into $\bar{\mu}\mu$ and $\bar{\tau}\tau$ is kinematically forbidden. If the active neutrino mixed with $\nu_s$ is either $\nu_\mu$ or $\nu_\tau$, then we can translate the mixing angle into lifetime as

$$
\tau_{\nu_s} \equiv \Gamma_{\nu_2}^{-1} = \frac{1.0\ \text{sec}}{(M_s/10\ \text{MeV})^5 \sin^2 2\theta}\,.
\tag{3}
$$

For the mixing with $\nu_e$ the numerator is 0.7 sec; it is is different because of the presence of charged-current interactions.

A sterile neutrino mixed with $\nu_a$ could be observed in direct experiments, in particular where upper bounds on neutrino masses are obtained (see the list of references in [7]). The most accurate limits of course exist for $\nu_e$ [8, 9], roughly $m_{\nu_e} < 3$ eV. However, these experiments cannot help much in eliminating a heavy sterile neutrino because they are not sensitive to the mass range $M_s > 10$ MeV which we consider. Such heavy neutrinos are simply not produced in beta-decays and their impact is only indirect, e.g. they renormalize vector and axial coupling constants.

The upper limit on the tau-neutrino mass obtained by the ALEPH Collaboration [10] could be translated into limits on mass/mixing of $\nu_\tau$ with $\nu_s$. However, one

would need to reanalyse the data under the assumption of two (or several) mixed neutrinos, taking account of the sensitivity to measure the energy spectrum for different values of $m_\nu$. The bound obtained in Ref. [11] based on the assumption that the average neutrino mass $\langle m \rangle = \cos^2 \theta\, m_1 + \sin^2 \theta\, m_2$ should be smaller than the experimental upper limit is too naive and can serve only as a rough order-of-magnitude estimate. Moreover the NOMAD bounds [2] are much more restrictive and thus we will not pursue the subject of the ALEPH bounds here.

## 3   Cosmological production and freeze-out of heavy sterile neutrinos

In the early universe sterile neutrinos are produced through their mixing with the active ones. The production rate for relativistic $\nu_s$ (i.e. for $T_\gamma \geq m_2$) can be approximately estimated as [12]:

$$\frac{\Gamma_s}{H} \approx \frac{\sin^2 2\theta_{\rm M}}{2} \left(\frac{T_\gamma}{T_w}\right)^3 , \tag{4}$$

where $H$ is the Hubble expansion parameter, $T_\gamma$ is the plasma temperature equal to the photon temperature. $T_w$ is the decoupling temperature of active neutrinos, taken to be 3 MeV, and $\theta_{\rm M}$ is the the mixing angle in the medium. According to the calculations of Ref. [13] one finds in the limit of small mixing:

$$\sin 2\theta_{\rm M} \approx \frac{\sin 2\theta}{1 + 0.76 \times 10^{-19}\, (T_\gamma/{\rm MeV})^6 (\delta m^2/{\rm MeV}^2)^{-1}} , \tag{5}$$

The matter effects become essential for

$$T_\gamma > 1.5 \times 10^3\, {\rm MeV} (\delta m^2/{\rm MeV}^2)^{1/6}. \tag{6}$$

For $\Gamma_s/H > 1$, sterile neutrinos were abundantly produced and their number density was equal to that of light active neutrinos, at least during some epoch. The production rate reaches a maximum at $T_{\rm max} = 1.28 \times 10^3 (\delta m^2/{\rm MeV}^2)^{1/6}$ MeV. For the masses that are considered below, $T_{\rm max}$ is well above the neutrino mass.

If the equilibrium number density of sterile neutrinos is reached, it would be maintained until $T_f \approx 4(\sin 2\theta)^{-2/3}$ MeV. This result does not depend on the heavy

neutrino mass because they annihilate with massless active ones, $\nu_2 + \nu_a \to all$. The heavy neutrinos would be relativistic at decoupling and their number density would not be Boltzmann suppressed if, say, $T_f > M_s/2$. This gives

$$\sin^2 2\theta (\delta m^2/\text{MeV}^2)^{3/2} < 500 \ . \tag{7}$$

If this condition is not fulfilled the impact of $\nu_s$ on BBN would be strongly diminished. On the other hand, for a sufficiently large mass and non-negligible mixing, the $\nu_2$ lifetime given in Eq. (3) would be quite short, so that they would all decay prior to the BBN epoch. (To be more exact, their number density would not be frozen, but follow the equilibrium form $\propto e^{-M_s/T_\gamma}$.)

Another possible effect that could diminish the impact of heavy neutrinos on BBN is entropy dilution. If $\nu_2$ were decoupled while being relativistic, their number density would not be suppressed relative to light active neutrinos. However, if the decoupling temperature is higher than, say, 50 MeV pions and muons were still abundant in the cosmic plasma and their subsequent annihilation would diminish the relative number density of heavy neutrinos. If the decoupling temperature is below the QCD phase transition the dilution factor is at most $17.25/10.75 = 1.6$. Above the QCD phase transition the number of degrees of freedom in the cosmic plasma is much larger and the dilution factor is approximately 5.5. However, these effects are essential for very weak mixing, for example the decoupling temperature would exceed 200 MeV if $\sin^2 2\theta < 8 \times 10^{-6}$. For such a small mixing the life-time of the heavy $\nu_2$ would exceed the nucleosynthesis time and they would be dangerous for BBN even if their number density is 5 times diluted.

Sterile neutrinos would never be abundant in the universe if $\Gamma_s/H < 1$. In fact we can impose a stronger condition demanding that the energy density of heavy neutrinos should be smaller than the energy density of one light neutrino species at BBN ($T \sim 1$ MeV). Taking into account a possible entropy dilution by factor 5 we obtain the bound:

$$\left( \delta m^2/\text{MeV}^2 \right) \sin^2 2\theta < 2.3 \times 10^{-7} \ . \tag{8}$$

Parameters satisfying this conditions cannot be excluded by BBN. A more detailed

consideration permits one to impose a somewhat better limit, but we will not go into such detail here.

Keeping these restrictions in mind we proceed to derive bounds on $M_s$ and the mixing angle $\theta$ demanding that the influence of heavy neutrinos on BBN should not be too strong.

# 4 Big-Bang Nucleosynthesis

In general, if a neutrino has a mass exceeding an MeV then it may influence the light element abundances unless it decays much before the onset of nucleosynthesis (see e.g. [14, 15] and references therein). A heavy unstable sterile neutrino will affect BBN in several ways. The main effect is through the increased energy density which leads to a faster expansion and hence an earlier freeze-out of the $n/p$-ratio. However, also the decay products must be taken into account since the electron neutrinos directly enter the $n$-$p$ reactions. Moreover, if the $\nu_s$ decays into the $e^+e^-$ channel the temperature evolution is altered (see [3, 16] for discussion).

The calculations are described in detail in Ref. [3]; here we only briefly describe the basic steps. First we introduce the dimensionless variables

$$x = m_0 a(t) \quad \text{and} \quad y = p\, a(t) \ , \tag{9}$$

where $a(t)$ is the cosmic scale factor and $m_0$ is an arbitrary normalisation factor that we have chosen as $m_0 = 1$ MeV. We also introduce a dimensionless temperature $T = T_\gamma/m_0$ and measure all masses in units of $m_0$. In terms of these variables the Boltzmann equations describing the evolution of the sterile neutrino, $\nu_s$, and the active neutrinos, $\nu_a$, can be written as

$$\partial_x f_{\nu_s}(x,y) = \frac{\left(f_{\nu_s}^{eq} - f_{\nu_s}\right)}{\tau_{\nu_s}/\text{sec}} \frac{1.48\,x}{(Tx)^2} \left(\frac{10.75}{g_*(T)}\right)^{1/2}$$
$$\left[\frac{M_s}{E_{\nu_s}} + \frac{3 \times 2^7 T^3}{M_s^3}\left(\frac{3\zeta(3)}{4} + \frac{7\pi^4}{144}\left(\frac{E_{\nu_s}T}{M_s^2} + \frac{p_{\nu_s}^2 T}{3E_{\nu_s}M_s^2}\right)\right)\right] \ , \tag{10}$$

$$\partial_x \delta f_{\nu_a} = \text{D}_a(x,y,M_s) + \text{S}_a(x,y) \ , \tag{11}$$

6

where $f_{\nu_s}^{eq} = (e^{E/T} + 1)^{-1}$, $\delta f_{\nu_a} = f_{\nu_a} - (e^y + 1)^{-1}$, and $E_{\nu_s} = \sqrt{M_s^2 + (y/x)^2}$ and $p_{\nu_s} = y/x$ are the energy and momentum of $\nu_s$. Further, $g_*(T) = \rho_{\text{tot}}/(\pi^2 T_\gamma^4/30)$ is the effective number of massless species in the plasma determined as the ratio of the total energy density to the equilibrium energy density of one bosonic species with temperature $T_\gamma$.

The scattering term in Eq. (11) comes from interactions of active neutrinos between themselves and from their interaction with electrons and positrons. For $\nu_e$ it has the form

$$
\begin{aligned}
S_{\nu_e}(x, y) \quad &= 0.26 \left(\frac{10.75}{g_*}\right)^{1/2} (1 + g_L^2 + g_R^2)(y/x^4) \\
&\times \left\{ -\delta f_{\nu_e} + \frac{2}{15} \frac{e^{-y}}{1 + g_L^2 + g_R^2} [1 + 0.75(g_L^2 + g_R^2)] \right. \\
&\quad \times \left[ \int dy_2 y_2^3 \delta f_{\nu_e}(x, y_2) + \frac{1}{8} \int dy_2 y_2^3 \left(\delta f_{\nu_\mu}(x, y_2) + \delta f_{\nu_\tau}(x, y_2)\right) \right] \\
&\quad \left. + \frac{3}{5}(T \cdot x - 1)\frac{g_L^2 + g_R^2}{1 + g_L^2 + g_R^2} e^{-y} (11y/12 - 1) \right\} , \quad (12)
\end{aligned}
$$

where $g_L = \sin^2\theta_W + \frac{1}{2}$, while $g_R = \sin^2\theta_W$. The corresponding term for $\nu_\mu$ and $\nu_\tau$ is given by Eq. (12) with the exchange $g_L \to \tilde{g}_L = \sin^2\theta_W - \frac{1}{2}$.

The decay term in Eq. (11) comes from the decay of heavy sterile neutrino according to reactions Eq. (2). This term depends on the mixing channel. For $\nu_\tau \leftrightarrow \nu_s$ mixings we have

$$
D_{\nu_e, \nu_\mu}(x, y) = \frac{467}{M_s^3 \tau_{\nu_s} x^2} \left(\frac{10.75}{g_*}\right)^{1/2} \left(1 - \frac{16y}{9M_s x}\right) \left(n_{\nu_s} - n_{\nu_s}^{eq}\right) \theta\left(M_s x/2 - y\right) , \quad (13)
$$

$$
\begin{aligned}
D_{\nu_\tau}(x, y) = \ & \frac{935}{M_s^3 \tau_{\nu_s} x^2} \left(\frac{10.75}{g_*}\right)^{1/2} \left[1 - \frac{16y}{9M_s x} + \frac{2}{3}\left(1 + \tilde{g}_L^2 + g_R^2\right)\left(1 - \frac{4y}{3M_s x}\right)\right] \\
&\times \left(n_{\nu_s} - n_{\nu_s}^{eq}\right) \theta\left(M_s x/2 - y\right) , \quad (14)
\end{aligned}
$$

where $n_{\nu_s}(x)$ is the number density of $\nu_s$ and $\theta(y)$ is the step function which ensures energy conservation in the decay. For $\nu_\mu \leftrightarrow \nu_s$ mixing these terms come from Eq. (13) and Eq. (14) by exchange of $\nu_\mu \leftrightarrow \nu_\tau$.

For $\nu_e \leftrightarrow \nu_s$ mixing those terms are

$$
D_{\nu_\tau, \nu_\mu}(x, y) = \frac{341}{M_s^3 \tau_{\nu_s} x^2} \left(\frac{10.75}{g_*}\right)^{1/2} \left(1 - \frac{16y}{9M_s x}\right) \left(n_{\nu_s} - n_{\nu_s}^{eq}\right) \theta\left(M_s/2 - y/x\right), \quad (15)
$$

$$D_{\nu_e}(x, y) = \frac{683}{M_s^3 \tau_{\nu_s} x^2} \left(\frac{10.75}{g_*}\right)^{1/2} \left[1 - \frac{16y}{9M_s x} + \frac{2}{3}\left(1 + g_L^2 + g_R^2\right)\left(1 - \frac{4y}{3M_s x}\right)\right]$$
$$\times \left(n_{\nu_s} - n_{\nu_s}^{\text{eq}}\right)\theta\left(M_s/2 - y/x\right) , \tag{16}$$

More details on the derivation of Eqs. (10,11) can be found in Ref. [3].

The temperature evolution is governed by the equation for energy conservation:

$$x\frac{d\rho}{dx} = -3(\rho + p) , \tag{17}$$

where $\rho$ and $p$ are energy density and pressure.

For the solution of Eqs. (10,11,17) we divide the time region into 3 parts. At early times, when the light neutrinos are kept in tight equilibrium with the electromagnetic plasma, we need only to solve the equation for the heavy sterile neutrinos, Eq. (10). At intermediate temperatures we solve the full set of equations, taking into account the changed temperature evolution and the non-equilibrium distribution functions of the neutrinos. At late times, long after the neutrinos have decoupled and the heavy neutrinos have disappeared, we solve only the kinetic equations governing the $n$-$p$ reactions needed for the nucleosynthesis code. We use a 800 point grid in the momentum region big enough to encompass the decay products $(0 < y < M_s x/2)$, and for the BBN calculations we use $\eta_{10} = 5$.

The results of the calculations have been imported into the modified Kawano nucleosynthesis code [17], and the abundances of all light elements have been calculated. At each time step $x$ we find the corresponding photon temperature and total energy density. Furthermore we integrate the kinetic equation governing the $n/p$ evolution taking into account the distorted spectrum of $\nu_e$.

For a given mass, $M_s$, we calculate the helium abundance, $Y$, for various mixing angles (or lifetimes) and use the approximate formula $\Delta Y = 0.013 \, \Delta N$ to find the minimum allowed mixing angle (Fig. 1) or the maximum allowed lifetime (Fig. 2) for that mass. We do this for both $\Delta N = 0.2$ and 1.0. On the same figures we plot guide-the-eye lines, that are fitted in the form

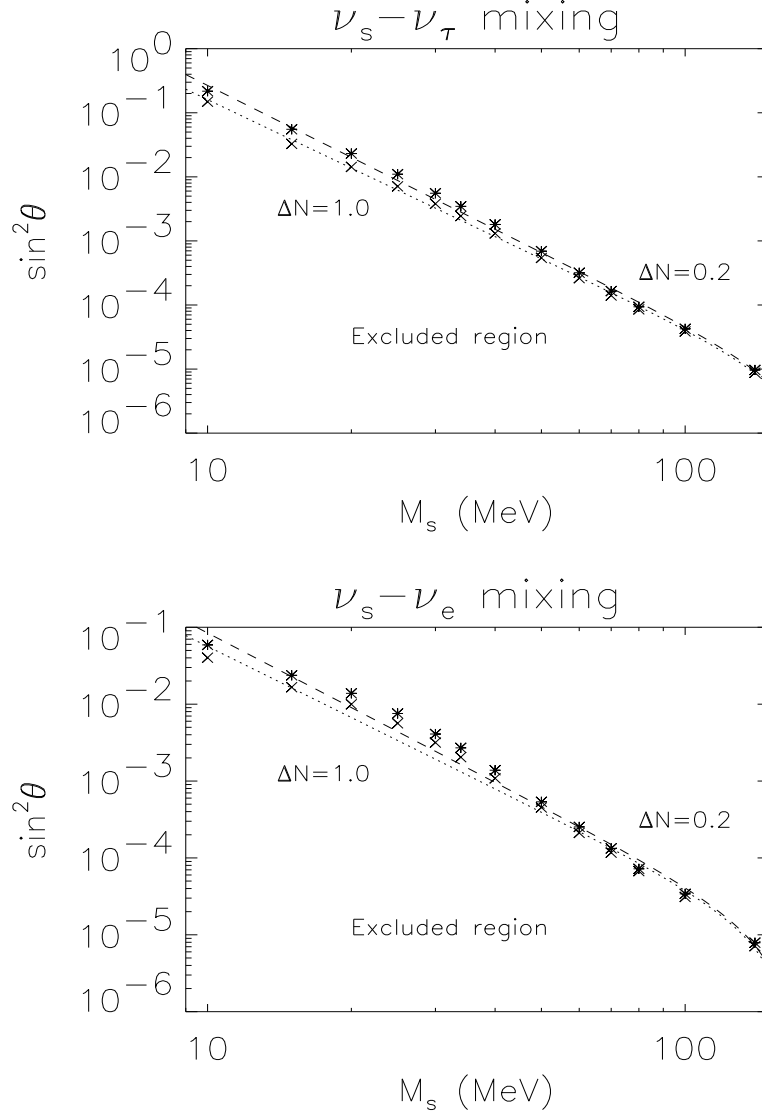$$(\sin^2\theta)_{\text{min}} = s_1 M_s^\alpha + s_2 \tag{18}$$

Figure 1: Minimum mixing angle between sterile and active neutrinos, allowed by BBN, as a function of the heavy neutrino mass, both for an optimistic bound, $\Delta N = 0.2$, and for a conservative bound, $\Delta N = 1.0$. The upper panel corresponds to $\nu_\mu \leftrightarrow \nu_s$ or $\nu_\tau \leftrightarrow \nu_s$ mixings, while the lower one to $\nu_e \leftrightarrow \nu_s$ mixing.
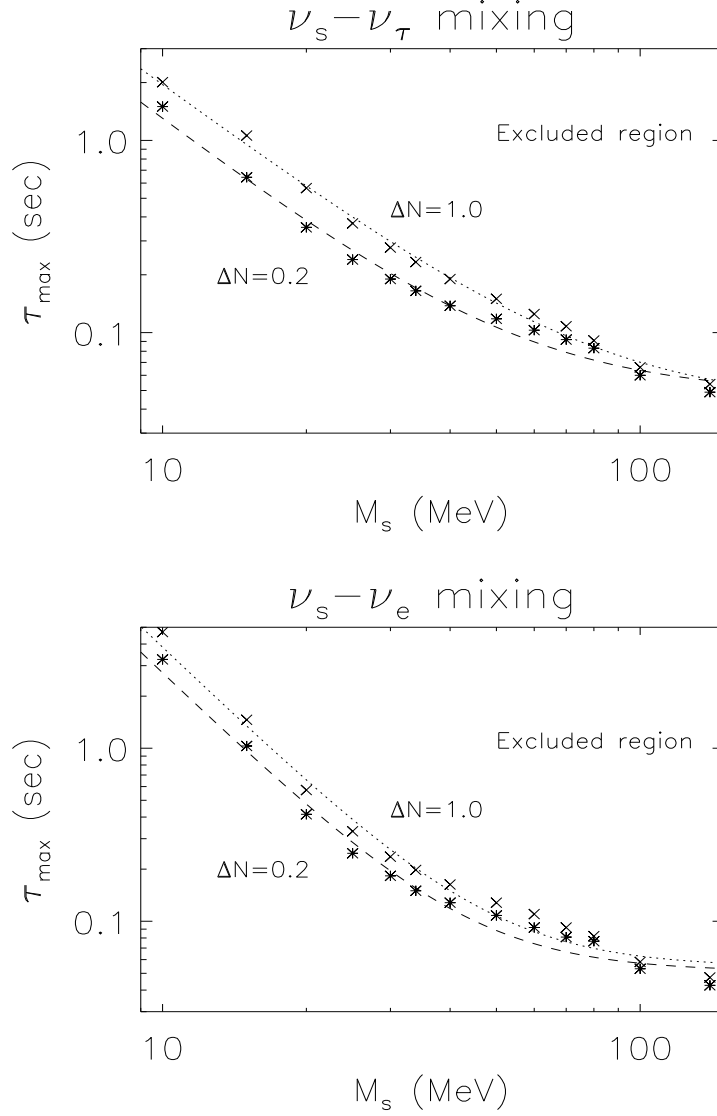
Figure 2: Maximum lifetime of sterile neutrino, allowed by BBN, as a function of the heavy neutrino mass, both for an optimistic bound, $\Delta N = 0.2$, and for a conservative bound, $\Delta N = 1.0$. The upper panel corresponds to $\nu_\mu \leftrightarrow \nu_s$ or $\nu_\tau \leftrightarrow \nu_s$ mixings, while the lower one to $\nu_e \leftrightarrow \nu_s$ mixing.

for the minimum allowed mixing and

$$\tau_{\max} = t_1 \, M_s^\beta + t_2 \qquad (19)$$

for the maximum allowed life-time $\tau_{\max}$. Fitting parameters in Eqs.(18,19) are presented at the Table 1.

| mixing | $\mathbf{\Delta N}$ | $(\sin^2\theta)_{\min}$ fitting parameters. | | | $\tau_{\max}$ fitting parameters. | | |
|---|---|---|---|---|---|---|---|
| | | $\mathbf{s_1}$ | $\mathbf{s_2}$ | $\alpha$ | $\mathbf{t_1}$ | $\mathbf{t_2}$ | $\beta$ |
| $\nu_s - \nu_{\mu,\tau}$ | 0.2 | 1423 | $-4.99 \cdot 10^{-6}$ | $-3.727$ | 100.1 | 0.04788 | $-1.901$ |
| $\nu_s - \nu_{\mu,\tau}$ | 1 | 568.4 | $-5.17 \cdot 10^{-6}$ | $-3.549$ | 128.7 | 0.04179 | $-1.828$ |
| $\nu_s - \nu_e$ | 0.2 | 140.4 | $-9.9 \cdot 10^{-6}$ | $-3.222$ | 1218 | 0.0513 | $-2.658$ |
| $\nu_s - \nu_e$ | 1 | 66.33 | $-1.05 \cdot 10^{-5}$ | $-3.070$ | 1699 | 0.0544 | $-2.652$ |

Table 1: Fitting parameters for minimum allowed mixing $(\sin^2\theta)_{\min}$ fitted by Eq.(18) and maximum allowed life-time $\tau_{\max}$ fitted by Eq.(19).

For small masses we followed Ref. [3] and used the expansion in a small parameter, $\Delta = T \cdot x - 1$, to describe the temperature evolution, whereas for bigger masses we used a more correct, and somewhat more complicated, treatment of the photon temperature from Eq. (17). The results are in perfect agreement for $M_s = 60$ MeV, and for $M_s = 140$ MeV we find the maximal allowed lifetime about 0.05 sec, in fair agreement with the approximate fitting formulae.

It is important to keep in mind, that the bounds obtained above are rather conservative, since all the approximations used in the derivation lead to slightly weakened bounds, as described in Ref. [3].

# 5  Decay $\nu_2 \to \nu_a + \pi^0$

The results obtained in the previous section are valid when the mass of the heavy neutrino is smaller than 140 MeV. For such low masses the dominant decay modes of $\nu_2$ would be into electrons and light neutrinos. A possible decay mode including muons which is possible for $M_s > 105$ MeV is suppressed if $M_s < 140$ MeV. However, for $M_s > 135$ MeV the decay channel

$$\nu_2 \to \pi^0 + \nu_a \tag{20}$$

becomes open and strongly dominating. The life-time of $\pi^0 \to \nu\bar\nu$ was calculated in refs. [18, 19]. We can translate their results for the decay (20) and find for the life-time

$$\tau = \left[\frac{G_F^2 M_s (M_s^2 - m_\pi^2) f_\pi^2 \sin^2\theta}{16\pi}\right]^{-1} = 5.8 \cdot 10^{-9} \sec \left[\sin^2\theta \frac{M_s(M_s^2 - m_\pi^2)}{m_\pi^3}\right]^{-1} \tag{21}$$

where $m_\pi = 135$ MeV is the $\pi^0$-mass and $f_\pi = 131$ MeV is the coupling constant for the decay $\pi^+ \to \mu + \nu_\mu$.

Immediately after $M_s$ becomes bigger than $m_\pi$ the two-body decay becomes the main one and all other channels can be neglected. We cannot directly apply our numerical program (that was made for a 33.9-MeV neutrino) to this case, so we will instead make some simple order of magnitude estimates for the impact of very heavy $\nu_2$ on BBN. One can roughly conclude that for the life-time of $\nu_2$ smaller than 0.1 sec, and corresponding cosmological temperature higher than 3 MeV, the decay products would quickly thermalize and their impact on BBN would be small. For a life-time larger than 0.1 sec, and $T < 3$ MeV, we assume that thermalization of neutrinos is negligible and approximately evaluate their impact on BBN. If $\nu_s$ is mixed with $\nu_\mu$ or $\nu_\tau$ then electronic neutrinos are not produced in the decay (20) and only the contribution of the decay products into the total energy density is essential. As we have already mentioned, non-equilibrium $\nu_e$ produced by the decay would directly change the frozen $n/p$-ratio. This case is more complicated and demands a more refined treatment than what is presented in this section.

The $\pi^0$ produced in the decay (20) immediately decays into two photons and they heat up the electromagnetic part of the plasma, while neutrinos by assumption are decoupled from it. We estimate the fraction of energy delivered into the electromagnetic and neutrino components of the cosmic plasma in the instant decay approximation. Let $r_s = n_s/n_0$ be the ratio of the number densities of the heavy neutrinos with respect to the equilibrium light ones, $n_0 = 0.09 T_\gamma^3$. The total energy of photons and $e^+e^-$-pairs including the photons produced by the decay is

$$\rho_{em} = \frac{11}{2}\frac{\pi^2}{30}T^4 + r_s n_0 \frac{M_s}{2}\left(1 + \frac{m_\pi^2}{M_s^2}\right), \tag{22}$$

while the energy density of neutrinos is

$$\rho_\nu = \frac{21}{4}\frac{\pi^2}{30}T^4 + r_s n_0 \frac{M_s}{2}\left(1 - \frac{m_\pi^2}{M_s^2}\right). \tag{23}$$

The effective number of neutrino species at BBN can be defined as

$$K_\nu^{(eff)} = \frac{22}{7}\frac{\rho_\nu}{\rho_{em}}. \tag{24}$$

Because of the stronger heating of the electromagnetic component of the plasma by the decay products, the relative role of neutrinos diminishes and $K_\nu^{(eff)}$ becomes considerably smaller than 3. If $\nu_s$ are decoupled while relativistic their fractional number would be $r_s = 4$ (two spin states and antiparticles are included). Possible entropy dilution could diminish it to slightly below 1. Assuming that the decoupling temperature is $T_d = 3$ MeV we find that $K_\nu^{(eff)} = 0.6$ for $M_s = 150$ MeV and $K_\nu^{(eff)} = 1.3$ for $M_s = 200$ MeV if the frozen number density of $\nu_s$ is not diluted by the later entropy release and $r_s$ remains equal to 4. If it was diluted down to 1, then the numbers would respectively change to $K_\nu^{(eff)} = 1.15$ for $M_s = 150$ MeV and $K_\nu^{(eff)} = 1.7$ for $M_s = 200$ MeV, instead of the standard $K_\nu^{(eff)} = 3$. Thus a very heavy $\nu_s$ would result in under-production of $^4He$. There could, however, be some other effects acting in the opposite direction.

Since $\nu_e$ decouples from electrons/positrons at smaller temperature than $\nu_\mu$ and $\nu_\tau$, the former may have enough time to thermalize. In this case the temperatures of $\nu_e$ and photons would be the same (before $e^+e^-$-annihilation) and the results obtained

above would be directly applicable. However, if thermalization between $\nu_e$ and $e^\pm$ was not efficient, then the temperature of electronic neutrinos at BBN would be smaller than in the standard model. The deficit of $\nu_e$ would produce an opposite effect, namely enlarging the production of primordial $^4He$, because it results in an increase of the $n/p$-freezing temperature. This effect significantly dominates the decrease of $K_\nu^{(eff)}$ discussed above. Moreover even in the case of the decay $\nu_2 \to \pi^0 + \nu_{\mu,\tau}$, when $\nu_e$ are not directly created through the decay, the spectrum of the latter may be distorted at the high energy tail by the interactions with non-equilibrium $\nu_\tau$ and $\nu_\mu$ produced by the decay. This would result in a further increase of $^4He$-production. In the case of direct production of non-equilibrium $\nu_e$ through the decay $\nu_2 \to \pi^0 + \nu_e$ their impact on $n/p$ ratio would be even much stronger.

To summarise, there are several different effects from the decay of $\nu_s$ into $\pi^0$ and $\nu$ on BBN. Depending upon the decay life-time and the channel these effects may operate in opposite directions. If the life-time of $\nu_2$ is larger than 0.1 sec but smaller than 0.2 sec, so that $e^\pm$ and $\nu_e$ establish equilibrium the production of $^4He$ is considerably diminished so that this life-time interval would be mostly forbidden. For life-times larger than 0.2 sec the dominant effect is a decrease of the energy density of $\nu_e$ and this results in a strong increase of the mass fraction of $^4He$. Thus large life-times should also be forbidden. Of course there is a small part of the parameter space where both effects cancel each other and this interval of mass/mixing is allowed. It is, however, difficult to establish its precise position with the approximate arguments described above. It would be a matter of separate and rather complicated non-equilibrium calculations.

Thus, in the case of $\nu_s \leftrightarrow \nu_{\mu,\tau}$ mixing for $M_s > 140$ MeV we can exclude the life-times of $\nu_s$ roughly larger than 0.1 sec, except for a small region near 0.2 sec where two opposite effects cancel and the BBN results remain undisturbed despite the presence of sterile neutrinos. Translating these results into mixing angle according to Eq.(21), we conclude that mixing angles $\sin^2 \theta < 5.8 \cdot 10^{-8} m_\pi/M_s/((M_s/m_\pi)^2 - 1)$ are excluded by BBN. Combining this result with Eq.(8) we obtain the exclusion

region for $M_s > 140 MeV$:

$$5.1 \cdot 10^{-8} \frac{\text{MeV}^2}{M^2} < \sin^2 \theta < 5.8 \cdot 10^{-8} \frac{m_\pi}{M_s} \frac{1}{(M_s/m_\pi)^2 - 1} \,. \tag{25}$$

In the case of $\nu_s \leftrightarrow \nu_e$ mixing for $M_s > 140 MeV$ the limits are possibly stronger, but it is more difficult to obtain reliable estimates because of a strong influence of non-equilibrium $\nu_e$ produced by the decay on neutron-proton reactions, and we will postpone this problem for future investigations.

# 6   Supernova 1987A

In Ref. [3] we have explained that massive sterile neutrinos can be constrained by the duration of the SN 1987A neutrino burst. If the sterile states live long enough and interact weakly enough to escape from the SN core, the usual energy-loss argument constrains the allowed interaction strength (in our case the active-sterile mixing angle). If the sterile states are so short lived or so strongly interacting that they decay or scatter before leaving the SN core, they contribute to the transfer of energy and thus still accelerate the cooling speed of the SN core.

The arguments given in Ref. [3] pertain directly to the present case as long as the sterile neutrino mass is not too large for its production to be suppressed in the relevant region of a SN core. Taking a typical temperature to exceed 30 MeV, the average thermal neutrino energy exceeds about 100 MeV so that threshold effects should not become important until the mass significantly exceeds this limit. Therefore, we believe that the SN limits pertain at least to masses up 100 MeV. For such a mass and maximal mixing, the lifetime is about 10 $\mu$s, for relativistic particles corresponding to a distance of about 3 km, i.e. of order the SN core radius, and much larger than the standard mean free path of active neutrinos of a few meters. Therefore, for the entire range of interesting masses and mixing angles the lifetime far exceeds what is necessary for the sterile neutrinos to transfer energy within the SN core in the spirit of the energy-transfer argument.

In summary, the arguments of Ref. [3] imply that for sterile neutrino masses below about 100 MeV the approximate range $3 \times 10^{-8} < \sin^2(2\theta) < 0.1$ is excluded.

# 7 Conclusion and Summary

In this work we have found cosmological and astrophysical bounds on possible mixings of a heavy sterile neutrino with mass 10 MeV $<$ $M_s$ $<$ 140 MeV with any active flavour $\nu_e$, $\nu_\mu$ or $\nu_\tau$. Our results are summarised in Fig. 3. The region between the two horizontal lines running up to 100 MeV are excluded by the duration of the neutrino burst from SN 1987A. A more detailed discussion would probably permit one to expand this region both in the horizontal and vertical directions.

The BBN limits are presented by the two upper dashed curves, both corresponding to the conservative bound of one permitted extra neutrino species. The curve for $\nu_{\mu,\tau}$ mixing is slightly higher than the curve for $\nu_e$ mixing. The lower dashed curve describes our approximate estimate of the efficiency of sterile neutrino production in the early universe Eq. (8). Below this curve the heavy $\nu_2$ are very weakly produced and their impact on BBN is not essential. Let us note that relation Eq. (7) is fulfilled for the entire BBN-excluded region of Fig. 3. In other words, sterile neutrinos are relativistic at decoupling and the Boltzmann suppression factor is not essential.

We have also made an order of magnitude estimates of the BBN-influence of heavier neutrinos with masses 140 MeV $<$ $M_S$ $<$ 200 MeV for $\nu_s \leftrightarrow \nu_{\mu,\tau}$ mixing channel. We have found, that mixing angles in the range given by Eq.(25) are excluded by BBN. We do not present these order of magnitude estimates in our Fig. 3 since they are less precise than the limits presented for $M_s <$ 140 MeV. We believe that the BBN bounds also could be improved considerably with more detailed calculations. For the $\nu_s \leftrightarrow \nu_\tau$ mixing a part of the $\sin^2 \theta$-$M_s$-plane is excluded by the NOMAD experiment [2]. Their bound is also presented in Fig. 3.

We have done the calculations for the Dirac neutrinos. For the Majorana case the cosmological number density of the heavy neutrinos would be twice smaller and corresponding bounds would be changed in an evident way and rather weakly.

It is noteworthy that the impact of sterile neutrinos on BBN is very sensitive to the deviation from thermal equilibrium of both sterile neutrinos as well as their decay products, especially into the $\nu_e$-channel. In the usual approximate calculations,
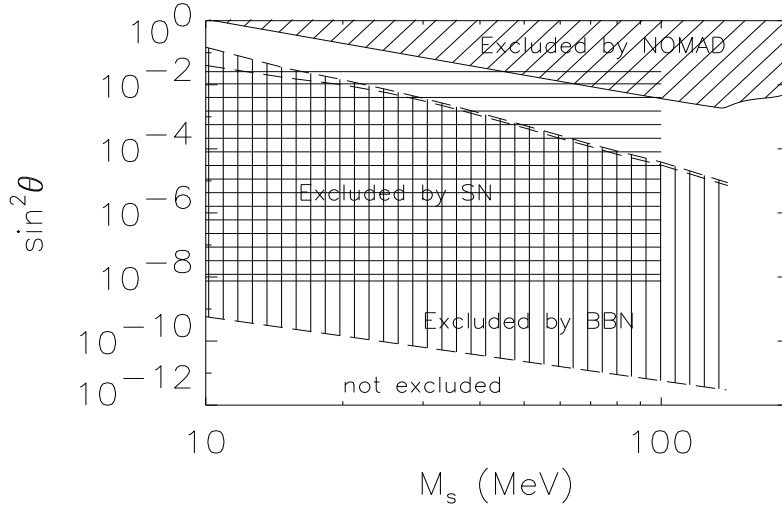
Figure 3: Summary of our exclusion regions in the $\sin^2\theta$-$M_s$-plane. SN 1987A excludes all mixing angles between two solid horizontal lines. BBN excludes the area below the two upper dashed lines if the heavy neutrinos were abundant in the early universe. These two upper dashed lines both correspond to the conservative limit of one extra light neutrino species permitted by the primordial $^4$He-abundance. The higher of the two is for $\nu_{\mu,\tau}$ mixing, and the slightly lower curve is for $\nu_e$ mixing. In the region below the lowest dashed curve the heavy neutrinos are not efficiently produced in the early universe and their impact on BBN is weak. For comparison we have also presented here the region excluded by NOMAD Collaboration [2] for the case of $\nu_s \leftrightarrow \nu_\tau$ mixing.

kinetic equilibrium is assumed. This assumption very much simplifies the calculations because the integro-differential kinetic equations are reduced to simple ordinary differential equations. However, we have found that this approximation deviates substantially from the more exact treatment presented here. Even more accurate calculations are feasible with the technique developed in our previous works [14, 15, 16] where an accuracy at the sub-percent level was achieved, but this is a difficult and time-consuming problem. We would like to postpone it until the existence of heavy sterile neutrinos becomes more plausible.

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